

Hörmander's theorem for path-dependent SDEs

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We establish the existence of smooth densities for solutions to a broad class of path-dependent SDEs under Hörmander's condition. Our coefficients at time t are allowed to depend on the whole history of the solution path up to t arbitrarily. To formulate Hörmander's bracket condition, which holds in the finite-dimensional state space for the given equation, we define the Lie brackets in terms of vertical derivatives in the sense of the functional Itô calculus. Our concept of the Lie bracket naturally extends the classical case. The approach we take relies on an interplay between the analysis of SDEs in Banach spaces, Malliavin calculus, and rough path techniques.