Controllability of affine control systems

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Abstract

In the early of 1980's B. Bonnard, V. Jurdjevic, I. Kupka and G. Sallet (see [1], [2] and [3]) studied affine controllability in connection with linear controllability. They considered an affine control system of the form

$$\dot{x} = Ax + a + \sum_{i=1}^{m} u_i(t)(B_i x + b_i), x \in \mathbb{R}^n,$$

where $A, B_1, \ldots, B_m$ are $n \times n$ matrices, $a, b_1, \ldots, b_m$ are vectors in $\mathbb{R}^n$ and $u(t) = (u_1(t), \ldots, u_m(t))$ is a control function with values in some control constraint set $\Omega \subset \mathbb{R}^n$. Hence, the study of such systems was made via analysis of a family of vector fields in the semidirect product of a Lie subgroup (generated by the exponential of $A, B_1, \ldots, B_m$) and $\mathbb{R}^n$.

In the context of semigroups of Lie groups consider V be an $n-$dimensional real vector space. Denote by End$V$ the set of all linear endomorphism on V and by $\mathrm{Gl}(V)$ the set of all automorphisms on V. Consider H a subgroup of $\mathrm{Gl}(V)$ with transitive action on $V \setminus \{0\}$ and take the group $G = H \rtimes V$ given by semidirect product of H and V. Recall that the affine group operation is defined by $(g, v) \cdot (h, w) = (gh, v + gw)$ for all $(g, v), (h, w) \in G$. Let $\pi : G \to H$ be the canonical projection of the affine group on the Lie group H. We call affine action the natural action of G on V, $(g, v) \cdot w = gw + v$ with $(g, v) \in G$ and $w \in V$. The natural action of $\pi(G) = H$ on V is called linear action. Given a semigroup $S \subset G$, the affine action of S on V is said transitive if $Sx = V$ for all $x \in V$. Moreover, $v \in V$ is called fixed point under S if $Sv = \{v\}$. Hence in this context the above result is

Theorem Consider $G = H \rtimes V$ an affine group. Let $S \subset G$ be a connected semigroup with non-empty interior. Suppose that the linear action of $\pi(S)$ is transitive on $V \setminus \{0\}$ and that S has no fixed point. Then the affine action of S on $V \setminus \{0\}$ is transitive.

Also in the early eighties, Jurdjevic and Kupka considered an invariant control system on $\mathrm{Sl}(n, \mathbb{R})$ given by

$$\dot{g} = Ag + uBg,$$

with unrestricted controls and matrices on $\mathfrak{sl}(n, \mathbb{R})$ and then presented conditions on the matrices A and B ensuring that this system is controllable. The

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main idea was to show that the Lie saturate of such a system is the whole Lie algebra $\mathfrak{sl}(n, \mathbb{R})$.

Several years later, in [6] Rocio, O., Santana, A. and Verdi, M. considered the affine control systems of type

$$\dot{x} = Ax + a + uBx + ub \tag{1}$$

where $A, B \in \mathfrak{s}l(2, \mathbb{R})$, $a, b \in \mathbb{R}^2$ and $u \in \mathbb{R}$ and give conditions on (A, a) and (B, b) ensuring that the above system is controllable in $\mathbb{R}^2$.

The main idea of this talk is present an initial study of controllability of invariant affine control system via Lie saturated.

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References

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