Characteristic Classes and Quaternionic Flags

Daniel Cavalcante Oliveira*
IMECC
UNICAMP
Campinas, Brasil

Abstract

Given a fiber bundle over a base manifold, one often wonders if the fibers are glued in a trivial manner. To answer this question, we're going to take a look at the characteristic classes of these bundles. These are topological invariants that can be calculated in many ways and they measure how far from being trivial the bundle is. For references, see [3], [4] and [6] as they provide a solid ground about this topic.

After a mathematical excursion in the world of algebraic topology and fiber bundles, we're going to focus our attention to quaternionic flag varieties and use many tools of Lie theory to reduce the topological problem of calculating these classes to an algebraic problem using the root system associated to a Lie algebra (see [1] and [2] for a classical overview on this matter. The basics are covered in [4], 2nd volume). This is a joint work with professor Luiz A. B. San Martin.

References

[1] Bernstein, I.N., Gelfand, I.M. and Gelfand, S.I., Schubert Cells and Cohomology of the Spaces G/P. Russian Math. Surveys 28 (1973), 1-26.

[2] Borel, A.: Topics in the Homology Theory of Fibre Bundles. Lecture Notes in Mathematics (Springer) 36 (1967).

[3] Hatcher, A.: Vector Bundles and K-Theory.

[4] Kobayashi and Nomizu: Foundations of Differential Geometry.

Real Flag Manifolds.

[5] L. A. B. San Martin: Algebras de Lie.

[6] Milnor and Stasheff: Characteristic Classes.