On the geometry of smooth structures: Prescribing scalar curvature on fiber bundles

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Abstract

Since the discovery of exotic spheres in the seminal work of John Milnor (see [**?**]), i.e, manifolds that are homeomorphic but no diffeomorphic to standard spheres, a lot has been questioned about the admissible geometries of these manifolds. It is known, for instance, that every exotic sphere of dimension 7 carries a metric of non-negative sectional curvature (see [**?**] and [**?**]) and a lot has been done concerning metrics of positive Ricci curvature on these manifolds (see [**?, ?, ?, ?, ?, ?, ?**]). On the other hand, it its not known if there is an exotic sphere with a metric of positive sectional curvature and Hitchin proved that there are exotic spheres that do not even admit metrics of positive scalar curvature. This raises the question: *at which extent do the smooth structures determine geometry?* In this presentation we discuss the problem of prescribing scalar curvature on the total space of general fiber bundles with compact structure group. As applications we discuss which smooth functions are realizable as scalar curvature functions on some exotic spheres and bundles over exotic spheres, as well as on tori bundles. We also sketch some results and future works concerning the possibility of realizing G-invariant functions (where G is a compact and connected Lie group) as the scalar curvature of G-invariant Riemannian metrics. This is a joint work with Llohann Sperança.