Analisys Session

A bit more of the classical Brezis-Nirenberg problem.

Adilson E. Presoto*
(UFSCar - Brazil)

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Abstract.

In the well-known paper [1], Brezis and Nirenberg have proved that if $0 < \lambda < \lambda_1$ and $N \geq 4$, then

$$\begin{cases} -\Delta u = \lambda u + |u|^{2^*-2}u \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{P}$$

where $2^* = 2N/(N-2)$ and λ is a real parameter, has a positive solution. In the following years, Capozzi, Fortunato and Palmieri extended the result for all $\lambda > 0$ if $N \geq 5$ or $\lambda > 0$ and $\lambda \neq \lambda_k$ in the case $N = 4$. Here $0 < \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_j \leq \cdots$ denote the eigenvalues of $-\Delta$ in $H_0^1(\Omega)$.

In this talk, we will deal with the resonant case, i.e. when λ an eigenvalue, in 4 dimensions. In this situation, a solution can no longer be obtained by applying the usual linking theorem. In [2], the authors use a version of the symmetric mountain pass theorem to solve the problem for $N \geq 5$ and λ being an eigenvalue, due to technicalities the proof does not work if $N = 4$.

*Departamento de Matemática, UFSCar, SP, Brasil, aepresoto@ufscar.br

It seems to us that the main difficult to apply a minimax theorem is to gettting the right estimates. For instance, we are not able to get a separation condition between the values over the boundary of a rectangle and a sphere in the finite-dimension subspace in order to apply the classical linking theorem. To overcome this, we use a variant of the linking theorem, where this condition is relaxed. We point out that this generalization was motivated by the orthogonalization technique introduced by Gazzolla and Ruf [3], and some variant of the classical minimax theorems without the separation condition obtained by Silva [4].

References

[1] H. Brezis and L. Nirenberg, *Positive solutions of nonlinear elliptic equations involving critical Sobolev expoents*, Comm. Pure Appl. Math. 36 (1983), 437–477.

[2] A. Capozzi, D. Fortunato, G. Palmieri, *An existence result for nonlinear elliptic problems involving critical Sobolev exponent*, Ann. Inst. H. Poincaré A.N.L **2** (1985), 463–470.

[3] F. Gazzola and B. Ruf, *Lower-order perturbations of critical growth nonlinearities in semilinear elliptic equations*, Adv. Diff. Eq. **2** (1997), 555–572.

[4] E.A. Silva, *Linking theorems and applications to semilinear elliptic problems at re-soance.* Non. Anal. 16, (1991), 455-477.