Number Theory Session

IMPROVEMENTS TO WARNING'S THEOREM

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Abstract.

Let $\mathbb{F}_q$ be the finite field with q elements, where q is a power of the characteristic p, and let $f = \{f_1, \cdots, f_r\}$ be a set of polynomials in $\mathbb{F}_q[x_1, \cdots, x_n]$ of degrees $d_1, \cdots, d_r$, respectively. Let $N(f, \mathbb{F}_q)$ denote the number of common zeros of $f_1, \cdots, f_r$ defined over $\mathbb{F}_q$.

The theorems due to Chevalley and Warning state that if $N(f, \mathbb{F}_q) > 0$ and if $n > d_1 + \cdots + d_r$, then $N(f, \mathbb{F}_q)$ is divisible by p and $N(f, \mathbb{F}_q) \geq q^{n-(d_1+\cdots+d_r)}$.

There are examples where $N(f, \mathbb{F}_q) = q^n - (d_1 + \cdots + d_r)$ under the stated hypotheses, showing that this bound is best possible.

Heath-Brown proved that if $N(f, \mathbb{F}_q) = q^n - (d_1 + \cdots + d_r)$, then the set of common zeros of f over $\mathbb{F}_q$ form an affine linear space (a subspace or a translate of a subspace).

Heath-Brown studied systems f where the set of common zeros of f over $\mathbb{F}_q$ does not form an affine linear space. In these cases, then subject to the above hypotheses, he showed that $N(f, \mathbb{F}_q) > q^n - (d_1 + \cdots + d_r)$ and he gave some lower bound estimates for $N(f, \mathbb{F}_q)$, which improve the classical estimate of $q^n - (d_1 + \cdots + d_r)$. In this talk I will present theorems that improve the estimates of Heath-Brown.

JOINT WORK WITH RACHEL PETRIK.

Keywords: Homogeneous Forms, Finite Fields, Number of Zeros.

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