Number Theory Session

PAIRS OF QUADRATIC FORMS OVER P-ADIC FIELDS.

DAVID LEEP*
(DEPARTMENT OF MATHEMATICS UNIVERSITY OF KENTUCKY
LEXINGTON, KENTUCKY, USA)

Thursday, February 08, 2024.
16h40 - 17h20

Online

Abstract.

Let K be a p-adic field and let $\mathcal{Q}_1, \mathcal{Q}_2 \in K[x_1, \cdots, x_n]$ be quadratic forms in n variables with coefficients in K.

Heath-Brown proved that if $n = 8$, $\mathcal{Q}_1, \mathcal{Q}_2$ have a nontrivial common zero defined over K, and the variety defined by $\{\mathcal{Q}_1, \mathcal{Q}_2\}$ is nonsingular, then there exist $a, b \in K$, not both zero, such that $a\mathcal{Q}_1 + b\mathcal{Q}_2$ splits off at least 3 hyperbolic planes.

This rather technical theorem, whose proof is very long, was an important ingredient to Heath-Brown's proof that nonsingular pairs of quadratic forms in 8 variables defined over a number field satisfy the Hasse Principle. More concretely: Suppose that $\{\mathcal{Q}_1, \mathcal{Q}_2\}$ is a pair of quadratic forms in 8 variables defined over a number field F and assume that the variety defined by $\{\mathcal{Q}_1, \mathcal{Q}_2\}$ is nonsingular. If $\mathcal{Q}_1, \mathcal{Q}_2$ have a nontrivial zero defined over each nonarchimedean completion of F and also over each real completion of F (if any), then $\mathcal{Q}_1, \mathcal{Q}_2$ have a nontrivial zero defined over F.

*Email: LEEP@UKY.EDU

Heath-Brown's theorem on pairs of quadratic forms over p-adic fields raised many questions in my mind, including why his proof was so difficult. Also, it seemed that this result for $n = 8$ should be part of a bigger theorem for general values of n.

This talk will explore the situation for general values of n.

JOINT WORK WITH JOHN HALL.

Keywords: Pairs of Quadratic Forms, p-adic fields.