



## ANALISYS SESSION

### Sharp regularity for the obstacle problem for $p$ -Laplacian type equations and applications.

João Vitor da Silva\*  
(UNICAMP - Brazil)

Monday, February 05, 2024.  
14h50 - 15h40  
Anfiteatro 11

#### Abstract.

In this Lecture we show existence/uniqueness of weak solutions of an obstacle problem for a quasi-linear operator with unbounded source terms. In our results, we obtain sharp gradient estimates, namely,  $C_{loc}^{1,\alpha}(B_1)$  for the solution to an explicit and universal regularity exponent. Our results are relevant even for the simplest model case governed by the  $p$ -Laplacian with Hölder continuous coefficients

$$\begin{cases} \operatorname{div}(|\nabla u|^{p-2}\mathfrak{U})\nabla u = f(x), & \text{in } \{u > \varphi\} \cap B_1 \\ \operatorname{div}(|\nabla u|^{p-2}\mathfrak{U}(x)\nabla u) \leq f(x), & \text{in } B_1 \\ u(x) \geq \varphi(x), & \text{in } B_1 \\ u(x) = \varphi(x), & \text{on } \partial B_1, \end{cases}$$

where  $f \in L^q(\Omega)$  for  $q > n$  and  $q \geq \frac{p}{p-1}$  ( $1 < p < \infty$ ),  $\mathfrak{U} \in C^{0,\sigma}(\Omega, \mathbb{R}^{n \times n})$  (for some  $\sigma \in (0, 1]$ ) with  $\mathfrak{U}$  a  $(\lambda, \Lambda)$ -uniformly elliptic matrix, and  $\varphi \in C^{1,\beta}(\Omega) \cap \mathfrak{X}_{p,q}$ , for some

\*Partially supported by (CNPq-Brazil) under Grant No. 307131/2022-0 and FAEPEX-UNICAMP 2441/23 - PIND - Projetos Individuais (03/2023), e-mail: jdasilva@unicamp.br.

$\sigma \in (0, 1]$ ) with  $\mathfrak{U}$  a  $(\lambda, \Lambda)$ - uniformly elliptic matrix, and  $\varphi \in C^{1,\beta}(\Omega) \cap \mathfrak{X}_{p,q}$ , for some  $\beta \in (0, 1]$  where

$$\mathfrak{X}_{p,q} := \{v \in W^{1,p}(\Omega); \quad \operatorname{div} \mathfrak{a}(x, \nabla v) \in L^q(\Omega)\}.$$

For some specific scenarios, we show the non-degeneracy of solutions, which provides crucial information about the free boundary of solutions. Our regularity estimates improve and extend, to a certain extent, results previously obtained for the obstacle problem governed by the  $p$ -Laplacian with bounded source term (cf. [1] and [3]). Furthermore, we gave special emphasis to the study of the linear and non-homogeneous case, i.e.,  $p = 2$  and  $f \neq 0$ , which was not available in the literature and it plays a decisive role in analysing the non-linear case (cf. [2]). This is a joint work with Elzon C. Bezerra Júnior (UFCA) and Romário T. Frias (Unicamp).

## References

- [1] Andersson, J., Lindgren, E. and Shahgholian, H., Optimal regularity for the obstacle problem for the  $p$ -Laplacian. *J. Differential Equations* 259 (2015), no. 6, 2167-2179.
- [2] Caareli, L. A., and Kinderlehrer, D., Potential methods in variational inequalities. *J. Analyse Math.* 37 (1980), 285-295.
- [3] Rodrigues, J.F., Stability remarks to the obstacle problem for  $p$ -Laplacian type equations. *Calc. Var. Partial Differential Equations* 23 (2005), no. 1, 51-65.