Number Theory Session

FACTORIZATION THEORY VIA ADDITIVE COMBINATORICS.

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Abstract.

We will introduce the relationship between factorization theory (focused on non-uniqueness) and zero-sum problems in additive combinatorics. Often the elements of a ring (such as the ring of integers of a number field) or of a monoid H can be written in several different ways as a finite product of irreducibles. The *set of lengths* of $a \in H$, $L(a) = \{k \in \mathbb{N} \,|\, \exists\, u_1, \cdots, u_k \text{ irreducible s.t. } a = u_1 \cdots u_k\}$, and the system of *set of lengths*, $\mathcal{L}(H) = \{L(a) \,|\, a \in H\}$, are means of describing the non-uniqueness of factorization in H. It is conjectured that $\mathcal{L}(\mathcal{B}(G))$, where $\mathcal{B}(G)$ is the monoid of zero-sum sequences over an abelian group G, completely determines G except for a few small cases. Furthermore, for $k \in \mathbb{N}$, let $\mathcal{U}_k(H)$ be the set of all $m \in \mathbb{N}$ such that there are irreducible $u_1, \cdots, u_k, v_1, \cdots, v_m$ satisfying $u_1 \cdots u_k = v_1 \cdots v_m$. Define $\lambda_k(H) = min\mathcal{U}_k(H), \rho_k(H) = sup\mathcal{U}_k(H)$ (k-th elasticity of H) and $\rho(H) = sup\,\dfrac{\rho_k(H)}{k}$ (elasticity of H). It is known that $\lambda_k(H)$ can be written in terms of $\rho_k(H)$, which in turn

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can be lower and upper bounded by the Davenport constant of G, which is defined as the maximum length of a minimal zero-sum sequence. The sets $\mathcal{U}_k(H)$ are usually well structured (they look like arithmetic progressions). For example, $\mathcal{U}_k(H) = [\lambda_k(H), \rho_k(H)]$ if and only if the ideal class group G of H is finite abelian. This makes the k-th elasticity one of the most important invariants to describe the non-uniqueness of the factorization in H. We will present the main results and conjectures about $\rho_k(H)$.

Keywords: Factorization theory, Krull monoids, Davenport constant.