

NUMBER THEORY SESSION

INTEGRAL VALUES OF GENERATING FUNCTIONS OF RECURRENCE SEQUENCES.

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Abstract.

Suppose that a_0, a_1, \cdots is an integer sequence which satisfies a recurrence relation with constant coefficients, and let T(x) = f(x)/g(x) be its generating function, where f(x) and g(x) have no common factors in $\mathbb{Z}[x]$. In this talk, we study the problem of finding the rational values of x such that T(x) is an integer. We say that such a number is good for the sequence. Our first main result is that if g(x) has at least two different irreducible factors, or if g(x) has a single irreducible factor of degree at least 3, then the sequence has only finitely many good values. We also study sequences of the form $0, 1, \cdots$ for which the recurrence relation has order 2. Among other results, we show that under a mild condition on the recurrence relation, the sequence has infinitely many good values, and we give a constructive method to find all of them.

JOINT WORK WITH MICHAEL KNAPP AND ABÍLIO LEMOS.

Keywords: Generating function, Recurrence sequences, Diophantine equations.

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