XII Workshop on Nonlinear Differential Equations

Universidade de Brasília, UNB

BRASÍLIA, BRAZIL

mat.unb.br/wnde2023

11-15 SEPTEMBER 2023

Posters, Plenaries

Scientific Committee:
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Universidade de Brasília

Welcome

It is with great honor that we welcome you to the XII Workshop on Nonlinear Differential Equations which will take place at the University of Brasília, Brazil, on September 11-115, 2023.

This workshop is the twelfth edition of a series of meetings which started in Brazil in 1996, originated mostly from the scientific collaboration between Italy and Brazil. In all of them, we had the participation of mathematicians not only from Italian and Brazilian institutions, but also from many other countries in Europe, U.S.A. and Latin America.

We wish you a pleasent stay and that you like the congress

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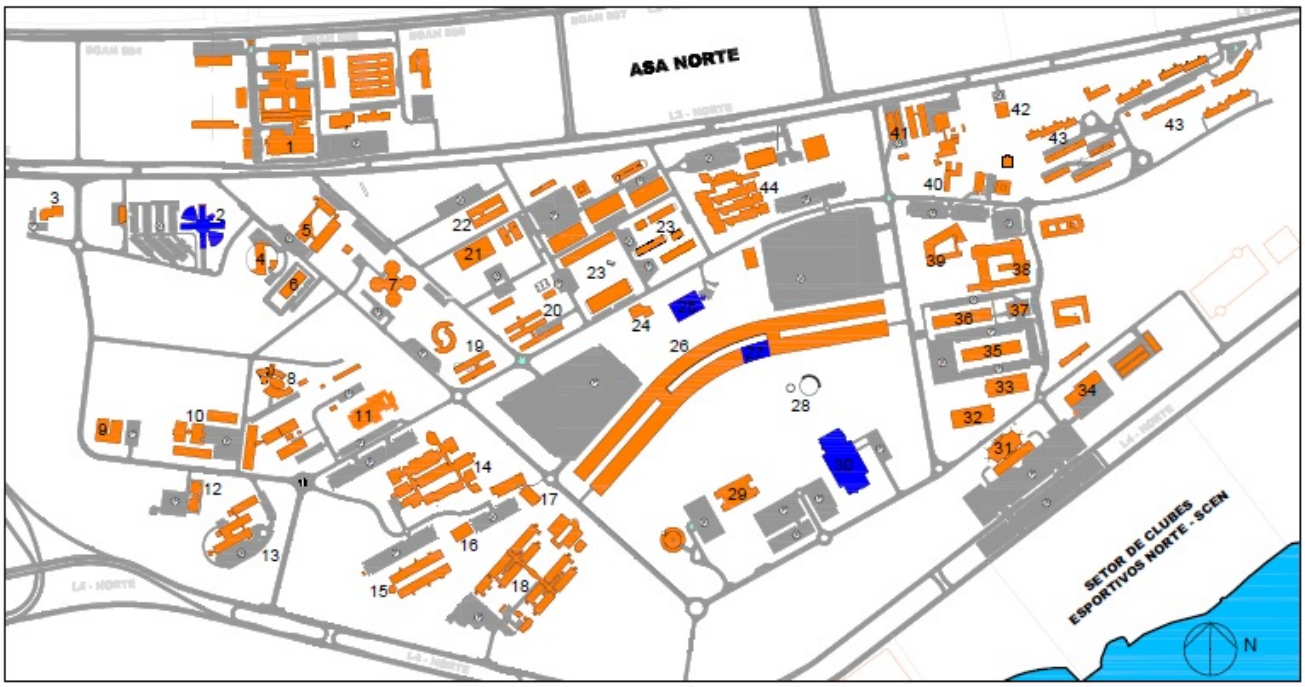
Map of the University of Brasília



1- HUB (Hospital Universitário) - University hospital	16- MASC	31- Centro Comunitário Athos Bulcão - Community Center
2- Finatec	17- BSA	32- CIC
3- FUBRA	18- Instituto de Biologia - Biology Institute	33- UED
4- AUTOTRAC	19- PMU II	34- Almoxarifado - Warehouse
5- FIO CRUZ	20- PMU II	35- PJC
6- CAEP	21- FE	36- PAT
7- CET	22- Casa do Professor	37- MASC
8- CRAD	23- SG 1 ao SG 12	38- FACE
9- CDT	24- Banco do Brasil - Bank	39- FA
10- CME	25- Restaurante Universitário - University Restaurant	40- ASFUB
11- NMT	26- ICC	41- Posto de Gasolina/Subway/Spoletto - Gas Station/Restaurant
12- CPD	27- Departamento de Matemática - Mth departament	42- PMDF - Police
13- CESPE	28- Teatro de Arena Honestino Guimarães - Arena Theater	43- Colina
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General Information

Location

The congress will take place in FINATEC at the University of Brasília. Also, the participant may want to know the departament of mathematics of the University of Brasília. Both places are indicated in blue in the map. Finatec is number 2 and the Department is number 27.

Meals and refreshments

We suggest the self-service restaurant at Finatec for lunch during the five days of the week of workshop. There is also a university restaurant indicated in the map (blue block near point 24). In this last level, there is a self-service restaurant serving lunch.

There are several restaurants nearby the in Asa Norte. We will present you a few options:

1. In front of the hotel HPlus Vision there is a shopping center with a food court containing many options (lunch and dinner)
2. Esquina Mineira located at SHCGN 704, bloco D - loja 42, Asa Norte (lunch).
3. Restaurante e Bar Xique Xique located at 708 Norte Bloco E Loja 45, Asa Norte (lunch and dinner).
4. Dona Lenha located CLN 413 Bloco D, Asa Norte (lunch and dinner).
5. Fratello Uni Pizzaria located at SHCN 109, Bloco D, Loja 19, Asa Norte (dinner)
6. in front of the hotel HPlus Vision there is a shopping center with a foor court
7. There is also a supermarket (Pão de Açucar) at 404/405 Norte, Bloco A, Asa Norte opened from 7:00hs to 22:00hs.

Social events

The Cocktail will be at the second floor in Finatec, on Monday, September 11th, at 7 pm. All the participants and friends are invited!

The Social Dinner will happen at restaurant Steak Bull Churrascaria. The menu serves vegetarians and non-vegetarians. There will be transportation from and back to the hotel HPlus Vision, leaving the hotel at 8pm and returning at 10pm.

Useful Phone Numbers

In case of any health emergencies call 192 (SAMU).

Police number: 190.

Math department of the University of Brasília: (61) 3107 6479

Security of the University of Brasília: (61) 3107 6222

Schedule

Schedule of the Workshop					
Time	Monday	Tuesday	Wednesday	Thursday	Friday
	11/09/2023	**12/09/2023**	**13/09/2023**	**14/09/2023**	**15/09/2023**
8h45-9h	**Opening**				
9h-9h40	F. Pacella	D. Araújo	M. Calanchi	M. Grossi	J. Rossi
9h45-10h25	D. Cassani	E. Teixeira	A. Quaas	J. D'Avila	B. Ruf
10h25 -10h50	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10h50 –11h30	D. Costa	C. Tarsi	B. Pellacci	B. Sirakov	F. Sani
11h35 –12h15	J. Bonder	O. Miyagaki	E. Santos	A. Saldaña	J. H. Andrade
12h15-14h00	Lunch	Lunch	Lunch	Lunch	Lunch
14h00–14h40	S. M. Soares	L. Iturriaga	J. Fernandes	L. Ferreira	M. Soares
14h45–15h25	G. Nornberg	J. Ratzkin	P. Ubilla	E. Medeiros	S. Nardulli
15h30 -16h10	D. Schiera	E. Terraneo		G. Romani	P. Piccione
16h10-16h40	Coffee Break & Poster Session 1	Coffee Break & Poster Session 2		Coffee Break & Poster Session 3	**Closing**
16h40-17h20	E. Parini	M. Pimenta		E. Topp	
17h25-18h05	E. Massa	J. Lopes		G. Siciliano	
19h	**Cocktail**				
20h			**Social Dinner**		

Poster Session 1: Juan P. Cabeza; Marcos Costa; Ignacio Dussel; Ranieri Freire

Poster Session 2: Raquel Lehrer; Jadde Oliveira; Raoni Ponciano; Gustavo Ramos

Poster Session 3: Sandra Neto; Ayana Santana; Claudemir S. Júnior; Aelson Sobral; Vitória Souza

Abstracts of the Talks

Classification results for critical sixth order PDEs with applications to conformal geometry

João Henrique Andrade (andradejh@ime.usp.br/andradejh@math.ubc.ca)
Universidade of São Paulo and University of British Columbia

Abstract. We are concerned with classifying entire positive singular solutions to a family of critical sixth order equations in the punctured space with a non-removable singularity at the origin. More precisely, we show that when the origin is a non-removable singularity, solutions are given by a singular radial factor times a periodic solution to a sixth order ODE with constant coefficients. On the technical level, we combine integral sliding methods and qualitative analysis of ODEs, based on the conservation of energy result, to perform a topological two-parameter shooting technique. Furthermore, we present applications of this classification theorem to study some classical problems in conformal geometry, namely compactness and bifurcation results for GJMS equations.

Joint work with João Marcos do Ó (UFPB), Jesse Ratzkin (Universität der Würzburg), Paolo Piccione (USP) and Juncheng Wei (University of British Columbia)

Infinity Laplacian equations with singular absorptions

Damião J. Araújo (araujo@mat.ufpb.br)
Universidade Federal da Paraíba

Abstract. In this talk, we discuss regularity aspects of singular free boundary problems governed by the infinity Laplacian. We explore existence of solutions, optimal growth at free boundary points, nondegeneracy properties, and fine geometric estimates for the free boundary.

Joint work with Ginaldo Sá (University of Central Florida)

References

[1] Araújo, D.J., Sá, G.S. Infinity Laplacian equations with singular absorptions. Calc. Var. 61, 132 (2022).

Integro-differential equations in Orlicz spaces: some recent results and open problems.

Julian Fernandez Bonder (jfbonder@dm.uba.ar)
Universidad de Buenos Aires

Abstract. In this talk I will present some recent results on regularity of weak solutions to nonlocal integro-differential equations in Orlicz spaces. More specifically, we show Holder continuity up to the boundary of weak solution of nonlocal versions of the $g-$Laplace operator with bounded right hand side.

Then we also show some regularity results for an eigenvalue problem associated to the same operator.

These results are contained in the articles [1] and [2].

Joint work with Ariel Salort (University of Buenos Aires) and Hernan Vivas (University of Mar del Plata)

References

[1] Fernández Bonder, Julián; Salort, Ariel; Vivas, Hernán. Interior and up to the boundary regularity for the fractional g-Laplacian: the convex case. Nonlinear Anal. 223 (2022), Paper No. 113060, 31 pp,

[2] Fernández Bonder, Julián; Salort, Ariel; Vivas, Hernán. Global Hölder regularity for eigenfunctions of the fractional g-Laplacian. J. Math. Anal. Appl. 526 (2023), no. 1, Paper No. 127332, 15 pp.

Remarks on singularity theory and geometry of some elliptic equations

Marta Calanchi (marta.calanchi@unimi.it)
Università degli Studi di Milano

Abstract.

The main objective of this talk is to study the structure of the set of solutions of some elliptic equations of the form

$$\begin{cases} -\Delta u - f(u) &= h(x) \qquad \text{in}\Omega \\ \frac{\partial u}{\partial \nu} &= 0 \qquad \text{on}\partial\Omega \end{cases}$$

using *Singularity Theory*.

A first paradigmatic example is given by the celebrated Ambrosetti-Prodi theorem: if the nonlinearity f is convex, and f' has interaction only with the first eigenvalue of the Laplacian, the map $\Phi(u) = -\Delta u - f(u)$ between appropriate functional spaces is a *global fold*.

We present some examples of nonlinearities f, whose derivative interacts only with a simple eigenvalue for which the singular set of Φ has only fold and cusp points...

This is a work in progress with B. Ruf.

Maximum principle for higher order operators in general domains and any dimension

Daniele Cassani (daniele.cassani@uninsubria.it)
Università degli Studi dell'Insubria and Riemann International School of Mathematics

Abstract. We discuss a general principle of perturbing higher order operators with lower order derivatives in order to restore the maximum principle in the framework in which it is well known to fail. This is somehow delicate and the main ingredient is a new Harnack-type inequality. We first prove De Giorgi type level estimates for functions in $W^{1,t}(\Omega)$, $\Omega \subset \mathbb{R}^N$, with $t > N \geq 2$. This augmented integrability enables us to establish a new Harnack type inequality for functions which do not necessarily belong to De Giorgi's classes as obtained by Di Benedetto-Trudinger for functions in $W^{1,2}(\Omega)$. As a consequence, we prove the validity of the strong maximum principle for uniformly elliptic operators of any even order, in fairly general domains and in any dimension, provided either lower order derivatives or inertial effects are taken into account.

References

[1] D. Cassani and A. Tarsia, *Maximum principle for higher order operators in general domains*, Adv. Nonlinear Anal. **11** (2022), 655–671.

[2] D. Cassani, C.C. Polvara and A. Tarsia, *Maximum principle for higher order elliptic operators with inertia in general domains and any dimension*, Preprint 2023.

On Radially Elliptic Equations with Critical Nonlinearities in $\mathbb{R}^2$

David Costa (david.costa@unlv.edu)
University of Nevada Las Vegas

Abstract. This is a continuation of our recent study about the effect of heterogeneity on existence and multiplicity results for elliptic PDEs with critical nonlinearities à la Trudinger-Moser in dimension $N = 2$. Our results are ultimately connected to the increase on the threshold of compactness that can be achieved in the Trudinger-Moser inequality under radial symmetry and in the presence of rapidly vanishing radial weight in $H^1_{0,r}(B)$, where B is the unit ball in $\mathbb{R}^2$. Such results are generalization to dimension $N = 2$ of the seminal results of W.-M. Ni in dimension $N = 3$, and of Adimurthi for bounded domains in dimension $N = 2$, as well as of many other authors, under strong uniform assumptions.

Joint work with Hossein Tehrani (University of Nevada Las Vegas)

References

[1] Heterogeneous Radially Semilinear Elliptic Equations with Critical Nonlinearities in $\mathbb{R}^2$, *Nonlinear Analysis - Real World Applications, 2023.*

An overdetermined elliptic problem in the theory of water waves

Juan D'Avila (jddb22@bath.ac.uk)
University of Bath UK

Abstract. We construct solutions to an overdetermined elliptic problem, which gives overhanging solitary waves with constant vorticity. Although there is numerical evidence for their existence, these almost singular solutions seem difficult to construct using complex variables or bifurcation theory. We employ a method that is similar in spirit to the desingularisation of constant mean curvature surfaces. This is collaboration with Manuel del Pino, Monica Musso and Miles Wheeler (U. of Bath).

The Nehari manifold for a degenerate logistic parabolic equation

Juliana Fernandes (jfernandes@im.ufrj.br)
Universidade Federal do Rio de Janeiro

Abstract. We analyse the behavior of solutions to a degenerate logistic equation with a nonlinear term of the form $b(x)f(u)$, where the weight function b is assumed to be nonpositive. We exploit variational techniques and comparison principle in order to study the evolutionary dynamics. A crucial role is then played by the Nehari manifold, as we note how it changes as the parameter λ in the equation or the function b vary, affecting the existence and non-existence of stationary solutions. We describe a detailed picture of the positive dynamics and also address the local behavior of solutions near a nodal equilibrium, which sheds some further light on the study of the evolution of sign-changing solutions.

Joint work with Liliane Maia (UnB, Brazil)

On a localization-in-frequency approach for a class of elliptic problems with singular boundary data

Lucas C. F. Ferreira (lcff@ime.unicamp.br)
State University of Campinas

Abstract. We consider a class of nonhomogeneous elliptic equations in the half-space with critical singular boundary potentials and nonlinear fractional derivative terms. The forcing terms are considered on the boundary and can be taken as singular measure. Employing a functional setting and approach based on Littlewood-Paley decomposition and localization in frequency, we obtain results on solvability, regularity and symmetry of solutions.

Joint work with Dr. Wender S. Lagoin (State University of Campinas)

Critical points of semi-stable solutions of Laplace-Beltrami operator on surfaces

Massimo Grossi (massimo.grossi@uniroma1.it)
Sapienza Universita' di Roma, Italy

Abstract.

We consider semi-stable *positive* solutions of the problem

$$\begin{cases} -\Delta_g u = f(u) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Here M is a two-dimensional model space, which means $M = \mathbb{S}^2, \mathbb{R}^2$ or $\mathbb{H}^2$ with the corresponding standard metrics of constant curvature $1, 0, -1$, Δ_g is the Laplace-Beltrami operator and $\Omega \subset M$ is a *geodesically* convex set.
Under some suitable additional assumption we will show that the solution admits a *unique* critical point.

Joint work with Luigi Provenzano (Sapienza Universita' di Roma, Italy)

Semilinear elliptic equations involving nonlinearities with zeros

Leonelo Iturriaga (leonelo.iturriaga@usm.cl)
Universidad Técnica Federico Santa María

Abstract. In this talk we review some results concerning with the existence and multiplicity of positive solutions for semilinear elliptic problems resembling the following form

$$\begin{cases} -\Delta u = \lambda f(u), & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, $N \geq 3$, f is a locally Lipschitz function defined in $[0, +\infty)$, which is nonnegative with a positive zero, and λ is a positive parameter. We will also explore how these results can be extended to the fractional operators.

References

[1] S. Alarcón, L. Iturriaga, A. Quaas, *Existence and multiplicity results for Pucci's operators involving nonlinearities with zeros*, Calc. Var. Partial Differential Equations **45** (2012), 443–454.

[2] S. Alarcón, L. Iturriaga and A. Ritorto, *Nonnegative solutions for the fractional Laplacian involving a nonlinearity with zeros*, Manuscripta Math. **167** (2022), no. 1-2, 345–363.

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[4] A. Ambrosetti, P. Hess, *Positive solutions of asymptotically linear elliptic eigenvalue problems*, J. Math. Anal. Appl. **73** (1980), 411–422.

[5] B. Barrios, J. García-Melián, L. Iturriaga, *Semilinear elliptic equations and nonlinearities with zeros*, Nonlinear Anal. **134** (2016), 117–126.

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[11] E. De Giorgi, *Sulla differenziabilità e l'analiticità delle estremali degli integrali multipli regolari*, Mem. Accad. Sci. Torino. Cl. Sci. Fis. Math. Nat. **3** (1957), 25–43.

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[16] Kouhestani, N., Mahyar, H., Moameni, A., *Multiplicity results for a non-local problem with concave and convex nonlinearities*, Nonlinear Analysis **182** (2019), 263–279.

[17] P. L. Lions, *On the existence of positive solutions of semilinear elliptic equations*, SIAM Review **24** (1982), 441–467.

[18] G. Molica Bisci, V. Radulescu, R. Servadei, Variational Methods For Nonlocal Fractional Problems. With a Foreword by Jean Mawhin, Encyclopedia of Mathematics and its Applications, Cambridge University Press 162, Cambridge, 2016.

[19] J. Moser, *A new proof of De Giorgi's theorem concerning the regularity problem for elliptic differential equations*, Comm. Pure Appl. Math. **13** (1960), 457–468.

[20] X. Ros-Oton, J. Serra, *The Pohozaev identity for the fractional Laplacian*, Arch. Ration. Mech. Anal. **213** (2014), 587–628.

[21] Xiaohui, Y., *Multiplicity solutions for fully nonlinear equation involving nonlinearity with zeros*, Commun. Pure Appl. Anal. **12** (2013), 451–459.

Existence of weak solutions for a nonhomogeneous incompressible cell-fluid Navier-Stokes model with chemotaxis

Juliana Honda Lopes (juhonlopes@gmail.com)
IME - USP

Abstract.

This work is concerned with the mathematical analysis of a general cell-fluid Navier-Stokes model with the inclusion of chemotaxis proposed by the authors in [1]. This general model relays on a mixture theory multiphase formulation. It consists of two mass balance equations and two general momentum balance equations, respectively, for the cell and fluid phase, combined with a convection-diffusion-reaction equation for oxygen. We investigate the existence of weak solutions in a two or three-dimensional bounded domain when the fluids are assumed to be incompressible with constant volume fraction.

Joint work with Gabriela Planas (IMECC-UNICAMP)

References

[1] Y. Qiao, S. Evje, A general cell-fluid Navier-Stokes model with inclusion of chemotaxis. *Math. Models Methods Appl. Sci* 30 (6) (2020) 1167-1215.

[2] J. H. Lopes, G. Planas, Existence of weak solutions for a nonhomogeneous imcompressible cell-fluid Navier-Stokes model with chemotaxis. *Math. Models Methods Appl. Sci* 46 (2023) 13695-13715.

Concave-convex behavior for a (stationary) Kirchhoff equation with degenerate and nonautonomous nonlocal coefficient

Eugenio Massa (eugenio@icmc.usp.br)
Universidade de São Paulo, Campus de São Carlos, Brazil.

Abstract. We study positive solutions for the Kirchhoff type equation

$$-M(x, \|u\|^2)\Delta u = \lambda f(u)$$

with Dirichlet boundary conditions in a bounded domain Ω, where $\|\cdot\|$ is the norm in $H_0^1(\Omega)$ and f, M are suitable functions.

The problem is nonvariational since the nonlocal coefficient M, possibly degenerate, depends on the point $x \in \Omega$. We show that these properties of M can produce interesting phenomena, even with simple homogeneous right hand sides, providing existence, nonexistence, and multiplicity results, due to the fact that the rate of growth with respect to u on the left hand side may change in Ω.

We consider some model examples are given, including one where M takes the form of the original Kirchhoff coefficient for the elastic string, but with nonhomogeneous material.

On a Hardy-Sobolev type inequality in the upper-half space and its applications

Everaldo Medeiros (everaldo@mat.ufpb.br)
Universidade Federal da Paraíba

Abstract. In this talk we present a Hardy-Sobolev type inequality in the upper-half space. This result is obtained by using a simple unified divergence approach. As an application we address some existence results for a class of quasilinear elliptic problem with Neumann/Robin boundary conditions.

Joint work with Emerson Abreu (Universidade Federal de Minas Gerais) and Marcelo Furtado (Universidade de Brasília)

References

[1] E. Abreu; M. Furtado; E. Medeiros, *On a Hardy-Sobolev inequality with a remainder term and applications*, (submetido).

On a class of fractional Hamiltonian system involving critical growth

Olimpio Hiroshi Miyagaki (olimpio@ufscar.br)
UFSCAR

Abstract. Consider a class of fractional hamiltonian systems with critical growth

$$\begin{cases} (-\Delta)^s u + V(x)u = K_2(x)g(v) + \lambda|v|^{2_s^*-2}v, & x \in \mathbb{R}^N \\ (-\Delta)^s v + V(x)v = K_1(x)f(u) + \lambda|u|^{2_s^*-2}u, & x \in \mathbb{R}^N \\ u, v > 0, \end{cases} \tag{1}$$

where $(-\Delta)^s$ stands for the fractional Laplacian with $s \in (0,1)$, $N > 2s$, V, K_1, $K_2 : \mathbb{R}^N \to \mathbb{R}$ are bounded continuous nonnegative functions, the nonlinearities $f,\ g : \mathbb{R} \to \mathbb{R}$ are nonnegative continuous functions, $\lambda \in \{0,\ 1\}$ and $2_s^* = \frac{2N}{N-2s)}$ is the critical Sobolev exponent for the fractional laplacian. Here f and g have a quasi-critical growth and the potential $V : \mathbb{R}^N \to \mathbb{R}$ can vanish at infinity depending on the behavior of the weight functions K_1 and K_2. The compactness results are overcame by following ideas of C.O. Alves and M.A.S. Souto (JDE 254, 2013) , more precisely, from Q. Han (arxiv.org/abs/1903.09059v1.) The talk is basead on the paper by M- -,Santana,Toon, Ubilla (Nonlinear Analysis 229, 2023).

Lusternik-Schnirelman and Morse theory for the Van der Waals-Cahn-Hilliard equation and system with volume constraint

Stefano Nardulli (stefano.nardulli@ufabc.edu.br)
Universidade Federal do ABC

Abstract. We give a multiplicity result for solutions of the Van der Waals-Cahn-Hilliard two-phase transition equation and multiple phase transition systems with volume constraints on a closed Riemannian manifold. Our proof employs some results from the classical Lusternik–Schnirelman and Morse theory, together with a technique, the so-called *photography method*, which allows us to obtain lower bounds on the number of solutions in terms of topological invariants of the underlying manifold. The setup for the photography method employs recent results from Riemannian clusters's isoperimetry for small volumes.

Joint works with João Henrique Andrade (IME-USP, São Paulo), Vieri Benci (Università di Pisa), Jackeline Conrado (UERJ, Rio de Janeiro) Dario Corona (Università di Camerino), Luis Eduardo Osorio Acevedo (Universidade Tecnica de Pereira, Colombia), Paolo Piccione (IME-USP, São Paulo).

Classification for fully nonlinear equations with Hardy potential

Gabrielle Nornberg (gnornberg@dim.uchile.cl)
University of Chile

Abstract. In this talk we discuss solvability and qualitative properties of positive radial solutions for a class of Lane-Emden type equations driven by Pucci extremal fully nonlinear operators with Henon-Hardy weights. We show how dynamical systems and energy techniques can be applied to classify solutions in radial domains, in addition to exhibit regularity and asymptotic that degenerate with respect to the weight.

Joint work with Liliane Maia (University of Brasilia) and Filomena Pacella (La Sapienza University of Rome).

Energy stability for semilinear elliptic problems

Filomena Pacella (pacella@mat.uniroma1.it)
University of Roma Sapienza

Abstract. Let Ω be a bounded domain contained in an unbounded region C in $\mathbb{R}^N$. We consider nondegenerate positive solutions of a semilinear elliptic problem in Ω and address the question of studying how the energy of such solutions varies with respect to perturbations of Ω inside C which preserve the volume. In the case when C is a cylinder we consider special one-dimensional solutions in domains Ω which are bounded cylinders, while when C is a cone we consider radial solutions in bounded spherical sectors. We study the stability of these domains, and respective solutions, as critical points of the energy functional and show that the fact that they locally minimize the energy (with respect to volume preserving perturbations of Ω)depends on the Neumann eigenvalues of the Laplace (or Laplace-Beltrami) operator on the domain which spans the cylinder or the cone.

Joint work with Danilo Gregorin Afonso (University of Roma Sapienza) and Alessandro Iacopetti (University of Torino)

Uniqueness of least energy solutions of the fractional Lane-Emden equation in the ball

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Abstract. A celebrated result of Gidas, Ni and Nirenberg [1] states that positive solutions of the Lane-Emden equation

$$\begin{cases} -\Delta u &= u^p & \text{in } B, \\ u &= 0 & \text{on } \partial B, \end{cases}$$

where $B \subset \mathbb{R}^N$ is a ball, and $1 < p < \frac{N+2}{N-2}$, are unique. It is still an open problem to fully extend this result to its fractional counterpart

$$\begin{cases} (-\Delta)^s u &= u^p & \text{in } B, \\ u &= 0 & \text{in } \mathbb{R}^N \setminus B, \end{cases}$$

where $0 < s < 1$, $1 < p < \frac{N+2s}{N-2s}$, and $(-\Delta)^s$ is the fractional laplacian.

In this talk we will show a partial result in this direction, namely, that uniqueness holds true in the restricted class of *least energy solutions* (which, in particular, are strictly positive - or strictly negative - in B).

Joint work with Azahara DelaTorre Pedraza (Università La Sapienza - Roma, Italy)

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Spectral optimization for weighted anisotropic problems

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Abstract. We will discuss some recent results concerning weighted eigenvalue problems in bounded Lipschitz domains $\Omega \subset \mathbb{R}^N$, $N \geq 1$, under Robin boundary conditions.
First, we will consider the presence of an anisotropic diffusion and we show the existence of two positive principal eigenvalues $\lambda^{\pm}$ respectively associated with a positive and a negative eigenfunction. We analyze the minimization of $\lambda^{\pm}$ with respect to the sign-changing weight, showing that the optimal eigenvalues $\Lambda^{\pm}$ are equal if the domain has a centre of symmetry and the optimal weights are of bang-bang type, namely piece-wise constant functions, each one taking only two values. As a consequence, the problem is equivalent to the minimization with respect to the subsets of Ω satisfying a volume constraint. The optimization problem is completely solved in one dimension, in the case of homogeneous Dirichlet or Neumann conditions, showing new phenomena induced by the presence of the anisotropic diffusion.
The analogous study in higher dimension is open in its generality even in the case of the Laplacian operator. With this respect, we will present some asymptotical results in the case of Neumann boundary conditions.
This kind of optmization problems naturally arises in the study of the optimal spatial arrangement of resources for a species to survive in an heterogeneous habitat.

Joint works with Dario Mazzoleni (Università di Pavia), Giovanni Pisante (Università della Campania "Luigi Vanvitelli"), Delia Schiera (Universidade de Lisboa) and Gianmaria Verzini (Politecnico di Milano).

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[3] Pellacci B., Pisante G., Schiera D., Spectral optimization for weighted anisotropic problems with Robin conditions. Preprint arXiv:2303.01401.

Invariant Minimal Surfaces via Global Bifurcation

Paolo Piccione (piccione.p@gmail.com)
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Abstract.

I will discuss the existence of multiple minimal spheres and tori using a symmetry reduction principle (Hsiang and Lawson), and global bifurcation theory (Rabinowitz)

On a quasilinear elliptic problem involving the 1-laplacian operator and a discontinuous nonlinearity

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Universidade Estadual Paulista - Unesp

Abstract. In this work, we study a quasilinear elliptic problem involving the 1-laplacian operator, with a discontinuous, superlinear and subcritical nonlinearity involving the Heaviside function $H(\cdot - \beta)$. Our approach is based on an analysis of the associated p-laplacian problem, followed by a thorough analysis of the asymptotic behaviour or such solutions as $p \to 1^+$. We study also the asymptotic behaviour of the solutions, as $\beta \to 0^+$ and we prove that it converges to a solution of the original problem, without the discontinuity in the nonlinearity.

Joint work with Gelson G.C. dos Santos and João R. dos Santos Júnior (UFPA)

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Large harmonic functions for fully nonlinear fractional operators

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Abstract. We study existence, uniqueness and boundary blow-up profile for fractional harmonic functions on a bounded smooth domain $\Omega \subset \mathbb{R}^N$. We deal with harmonic functions associated to uniformly elliptic, fully nonlinear nonlocal operators, including the linear case

$$(-\Delta)^s u = 0 \quad \text{in } \Omega,$$

where $(-\Delta)^s$ denotes the fractional Laplacian of order $2s \in (0,2)$. We use the viscosity solution's theory and Perron's method to construct harmonic functions with zero exterior condition in $\bar{\Omega}^c$, and boundary blow-up profile

$$\lim_{x \to x_0, x \in \Omega} \operatorname{dist}(x, \partial\Omega)^{1-s} u(x) = h(x_0), \quad \text{for all} \quad x_0 \in \partial\Omega,$$

for any given boundary data $h \in C(\partial\Omega)$. Our method allows us to provide blow-up rate for the solution and its gradient estimates. Results are new even in the linear case.

Joint work with Gonzalo Dávila and Erwin Topp.

Gluing constructions of singular constant Q-curvature metrics

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Abstract. The fourth order Q-curvature of a Riemannian metric is a scalar generalizing its scalar curvature. In many ways the theory of Q-curvature of conformal metrics mirrors that of the classical Yamabe problem, except that the leading term of the corresponding PDE is a power of the Laplacian. Thus it is natural to prove similar results in this higher–order setting despite the complications coming from a higher order PDE. I will discuss a construction of complete metrics with constant Q-curvature with finitely many point singularities, which is equivalent to solving a fourth-order, nonlinear, elliptic PDE with infinite boundary values.

Joint work with J. H. Adrade (U. São Paulo), R. Caju (U. Chile),
J. M. do Ò (F. U. Paraìba) and A. Silva Santos (F. U. Sergripe)

Nonlocal planar Schrödinger-Poisson systems in the fractional Sobolev limiting case

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Università degli Studi dell'Insubria, RISM

Abstract.

This talk will focus on existence and symmetry for a nonlocal planar Schrödinger-Poisson system driven by the s-fractional p-Laplacian with $p = \frac{2}{s}$. Since the dimension two is the limiting case for the Sobolev embedding of $W^{s,p}(\mathbb{R}^2)$, one can deal with exponential growths. We prove the existence of solutions by means of a variational approximating procedure for an auxiliary Choquard equation in which we uniformly approximate the sign-changing logarithmic kernel of the Poisson equation. Qualitative properties of solutions such as symmetry and decay are also established by exploiting a suitable moving planes technique.

Joint work with Daniele Cassani (Università degli Studi dell'Insubria, RISM) and Zhisu Liu (Wuhan University).

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[1] Cassani D., Liu L., Romani G. *Nonlocal planar Schrödinger-Poisson systems in the fractional Sobolev limiting case.* Preprint available at arxiv:2305.15274.

Convex functions and PDEs

Julio D. Rossi (jrossi@dm.uba.ar)
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Abstract. In this talk we will introduce different notions of convexity that interpolate between classical convexity and quasiconvexity and that, moreover, have a natural fractional extension. For these notions of convexity we also characterize the convex envelope inside a domain of a boundary datum in terms of being the unique viscosity solution to an associated equation.

Joint work with P. Blanc (Buenos Aires), L. Del Pezzo (Montevideo), A. Quass (Valparaiso) and M. Parviainen (Jyvaskyla).

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On an inequality of Bliss-Moser type

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Abstract. We derive a limiting inequality for the integral inequalities by Bliss. We then consider a critical version of this inequality which is of Moser type, and discuss related non-compactness properties. This is joint work with J.M. do Ó and Pedro Ubilla.

Applications of the logarithmic Laplacian in the analysis of fractional problems

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Universidad Nacional Autónoma de México (UNAM)

Abstract. Fractional derivatives are commonly used to model a variety of phenomena, but... what does it mean to have a logarithmic derivative? And what would it be used for?

In this talk we focus on the logarithmic Laplacian, a pseudodifferential operator that appears as a first order expansion of the fractional Laplacian $(-\Delta)^s$ as the exponent s goes to zero. This operator can also be represented as an integro-differential operator with a zero-order kernel.

We will survey some recent results on how the logarithmic Laplacian can be used to study the behavior of linear and nonlinear fractional problems in the small order limit. This analysis will also reveal a deep and interesting mathematical structure behind the set of solutions of Dirichlet logarithmic problems.

Joint work with Felipe Angeles (UNAM), Héctor Chang (Centro de Investigación en Matemáticas), Víctor Hernández-Santamaría (UNAM), Sven Jarohs (Frankfurt University), and Tobias Weth (Frankfurt University).

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Extremal functions for Adams inequalities with Navier boundary conditions

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Abstract. We consider the problem of existence of extremal functions for second order Adams' inequalities with Navier boundary conditions on balls in $\mathbb{R}^n$ in any dimension $n \geq 4$, see [1]. We also discuss some sharp weighted versions of Adams' inequality [2] on the same spaces. The weights that we consider determine a supercritical exponential growth, except in the origin, and the corresponding inequalities hold for spherically symmetric functions only.

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[2] F. Sani. *On weighted second order Adams inequalities with Navier boundary conditions.* Bruno Pini Mathematical Analysis Seminar **13** (2022), no. 1, 44–67.

Spectral partition problems with volume and inclusion constraints

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Abstract. In this talk we discuss a class of spectral partition problems with a measure constraint, for partitions of a given bounded connected open set. We establish the existence of an optimal open partition, showing that the corresponding eigenfunctions are locally Lipschitz continuous, and obtain some qualitative properties for the partition. The proof uses an equivalent weak formulation that involves a minimization problem of a penalized functional where the variables are functions rather than domains, suitable deformations, blowup techniques and a monotonicity formula.

Joint work with with Pêdra Andrade, Makson Santos and Hugo Tavares (IST-Lisboa).

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Existence of solutions on the critical hyperbola for a pure Lane-Emden system with Neumann boundary conditions

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Abstract. We will consider a Lane-Emden system on a bounded regular domain with Neumann boundary conditions and critical nonlinearities, and we show that, under suitable conditions on the exponents in the nonlinearities, least-energy (sign-changing) solutions exist, and they are classical. In the proof we exploit a dual variational formulation which allows to deal with the strong indefinite character of the problem, and we establish a compactness condition which is based on a new Cherrier type inequality. We then prove such condition by using as test functions the solutions to the system in the whole space and performing delicate asymptotic estimates.

Joint work with A. Pistoia (Sapienza Università di Roma) and H. Tavares (Instituto Superior Técnico, Universidade de Lisboa)

Critical points under the energy constraint: existence and bifurcation results

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Abstract.

We discuss the existence of critical points for a family of abstract and smooth functionals on Banach spaces under the energy constraint. In other words, for a family of functionals $\Phi_\mu : X \to \mathbb{R}$, given $c \in \mathbb{R}$, we look for solutions $(u, \mu) \in X \times \mathbb{R}$ of the system

$$\begin{cases} \Phi_\mu(u) = c \\ \Phi'_\mu(u) = 0. \end{cases}$$

By means of the Ljusternick-Schnirelmann theory we show, under suitable assumptions, multiplicity results. The abstract framework is then applied to some partial differential equations depending on a parameter for which we obtain multiple solutions as well as some bifurcation results.

Joint work with Kaye Silva (Universidade Federal de Goiás, BR) and Humberto R. Quoirin (Universidad Nacional de Córdoba, AG).

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Uniform a priori estimates for the Lane–Emden system in the plane

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Abstract. We prove that positive solutions of the superlinear Lane-Emden system in a two-dimensional smooth bounded domain are bounded independently of the exponents in the system, provided the exponents are comparable. As a consequence, the energy of the solutions is uniformly bounded, a crucial information in their asymptotic study. On the other hand, the boundedness may fail if the exponents are not comparable, a surprising incidence of a situation in which the sub-critical Lane-Emden system behaves differently from the scalar equation.

Joint work with Nikola Kamburov (PUC-Chile).

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Energy Estimates and Multiplicity of Seminodal Solutions to an Elliptic System with Mixed Couplings

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Abstract. We study the system of semilinear elliptic equations

$$-\Delta u_i + u_i = \sum_{j=1}^{\ell} \beta_{ij} |u_j|^p |u_i|^{p-2} u_i, \qquad u_i \in H^1(\mathbb{R}^N), \qquad i = 1, \dots, \ell,$$

where $N \geq 4$, $1 < p < \frac{N}{N-2}$, and the matrix (β_{ij}) is symmetric and admits a block decomposition such that the entries within each block are positive or zero and all other entries are negative.

We provide simple conditions on (β_{ij}), which guarantee the existence of fully nontrivial solutions, i.e., solutions all of whose components are nontrivial. We establish existence of fully nontrivial solutions to the system having a prescribed combination of positive and nonradial sign-changing components, and we give an upper bound for their energy when the system has at most two blocks.

We derive the existence of solutions with positive and nonradial sign-changing components to the system of singularly perturbed elliptic equations

$$-\varepsilon^2 \Delta u_i + u_i = \sum_{j=1}^{\ell} \beta_{ij} |u_j|^p |u_i|^{p-2} u_i, \qquad u_i \in H_0^1(B_1(0)), \qquad i = 1, \dots, \ell,$$

in the unit ball, exhibiting two different kinds of asymptotic behavior: solutions whose components decouple as $\varepsilon \to 0$, and solutions whose components remain coupled all the way up to their limit.

KEYWORDS: Nonlinear elliptic system, weakly coupled, mixed cooperation and competition, positive and sign-changing components, singularly perturbed elliptic system.

Joint work with Mónica Clapp (Universidad Nacional Autónoma de México)

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Semilinear elliptic problems in $\mathbb{R}^N$: the interplay between the potential and the nonlinear term

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Abstract. It is considered a semilinear elliptic partial differential equation in $\mathbb{R}^N$ with a potential that may vanish at infinity and a nonlinear term with subcritical growth. A positive solution is proved to exist depending on the interplay between the decay of the potential at infinity and the behavior of the nonlinear term at the origin. The proof is based on a penalization argument, variational methods, and L^∞-estimates. Those estimates allow dealing with settings where the nonlinear source may have supercritical, critical, or subcritical behavior near the origin. Results that provide the existence of multiple and infinitely many solutions when the nonlinear term is odd are also established.

Joint work with Elves Alves de Barros e Silva (Universidade de Brasília).

Mass weighted Trudinger-Moser type inequalities on the plane and applications

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Abstract. We discuss some recent results on Trudinger type inequalities on $\mathbb{R}^2$ with *increasing* radial weights, in the general setting of nonradial mass weighted Sobolev spaces. We also prove the validity of the corresponding sharp Moser inequality in the special case of the logarithmic weight. The increasing nature of the weights prevents the application of any symmetrization technique: our main tools will be a suitable change of variable and a careful analysis of the behaviour of the normalized maximizing sequences. We will also discuss some applications and open problems. .

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The Bernoulli problem with unbounded jumps

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Abstract. The Bernoulli problem appears naturally in mathematical models arising from fluid dynamics, cavitation, jet flows, optimal designs, to cite a view. The analysis of such models leads to a very rich class of free boundary problems. In this talk I will discuss Bernoulli free boundary problems prescribing unbounded jumps. The analysis is considerably more intricate as solutions are expected to be only Hölder continuous, rather than Lipschitz as in the classical theory. The proof of sharp regularity estimate and the analysis of fine geometric measure properties of the free boundary require several new ingredients. The ultimate goal is to classify the varying cups geometries along the free boundary.

Joint work with S. Snelson (Florida Institute of Technology)

Non-uniqueness for critical heat equations in two dimensions

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University of Milano, Italy

Abstract. In this talk we deal with a class of nonlinear heat equations in two dimensions. Recently, for some specific nonlinearities with exponential growth of Trudinger-Moser type, Ioku et al [1] and Ibrahim et al [2] exhibit a singular stationary solution. Then, they prove that the Cauchy problem, with this singular solution as initial data, admits, at least, two different solutions. Here we consider similar problems for a wider class of nonlinearities in two dimensions.

Joint work with Yohei Fujishima (Shizuoka University, Japan), Norisuke Ioku (Tohoku University, Japan) and Bernhard Ruf (Istituto Lombardo, Italy).

References

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A nonlocal version of the inverse problem of Donsker and Varadhan

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Abstract. In their influential article of 1976, Donsker and Varadhan [1] addressed the following "inverse problem": suppose there are given two operators L_1, L_2 with the form

$$L_i\phi = Div(A_i(x)D\phi) + b_i(x)\cdot D\phi, \quad i = 1, 2,$$

with smooth coefficients, such that

$$\lambda_1(L_1 + V, \Omega) = \lambda_1(L_2 + V, \Omega),$$

for every potential $V \in C^\infty(\mathbb{R}^d)$, and every bounded domain $\Omega \subset \mathbb{R}^d$. What can be said about the relation among L_1 and L_2? Here, $\lambda_1(L_i + V, \Omega)$ denotes the principal eigenvalue of $L_i + V$ on Ω, for $i = 1, 2$. It is proven in [1] that $A_1 = A_2$; and either $L_1\phi = L_2(u\phi)/u$ for some L_2-harmonic function u, or $L_1\phi = L_2^*(u\phi)/u$ for some L_2^*-harmonic function u, where L_2^* is the adjoint operator of L_2.

In this talk, we report a nonlocal a version of this problem, details of which can be found in [2]. We prove a similar conjugacy phenomena among fractional operators in the spirit of [1]. However, the program requires different techniques compared with the original problem to tackle the nonlocal nature of the operators. The result is new, even in the case of purely diffusive operators with constant coefficients.

Joint work with Gonzalo Dávila (UTFSM-Chile)

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Some Superlinear Elliptic Problems

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Abstract. One of the most studied problems in nonlinear PDEs in the last five decades refers to the existence of positive solutions of the following elliptic problem

$$\begin{cases} -\Delta u = f(x,u), & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega \end{cases} \tag{2}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and $f : \Omega \times \mathbb{R} \to \mathbb{R}^+$ is a nonlinearity satisfying a superlinear condition at ∞. We will present here several pioneering works in this line and some contributions made with some of my collaborators.

Abstracts of the Posters

Hölder Regularity Theory for Nonloncal Operators

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Universidad de Chile

Abstract.

In this poster we discuss some regularity results for viscosity solutions of fully nonlinear elliptic second order differential equations of nonlocal type.

We start recalling the classical Hölder regularity developed for linear operators, local and global estimates, as well as the respective Dirichlet problem in a bounded domain. Then we revisit several articles concerning the nonlocal theory that explore the incorporation of new variants into the problem, such as: changing from classical to viscosity sense; working with non smooth domains [1]; replacing the linear for a nonlinear structure [3], [4]; and dealing with gradient terms [2]. Finally, we show how to establish a general result which puts all those elements together, while highlighting the main difficulties in the process.

Joint work with Gabrielle Nornberg (Fcfm, Universidad de Chile).

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Unbounded Continuum of Coexistence States for a Nonlocal and Nonlinear Elliptic System

Marcos Antonio Viana Costa (marcos.viana@unesp.br)
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Abstract. In this work, we will present a result that provides the existence of a continuum of coexistence states for the following nonlocal and nonlinear elliptic system:

$$\begin{cases} -a\left(\int_\Omega v\right)\triangle u = \lambda u - u^2 + buv, & \text{in } \Omega \\ -\triangle v + \sigma v = \rho u, & \text{in } \Omega \\ u = v = 0, & \text{on } \partial\Omega \end{cases}, \tag{P}$$

where Ω is a bounded regular domain of $\mathbb{R}^N$, $N \geqslant 1$, $a : \mathbb{R} \to (0, +\infty)$ is a continuous increasing function, $b, \lambda \in \mathbb{R}$, $\rho \geqslant 0$ and $\sigma > 0$. In order to guarantee the existence of an unbounded continuum $\mathfrak{C}$ in $\mathbb{R}\times\left(C_0^1\left(\overline{\Omega}\right)\right)^2$ of coexistence states of (P) emanating from the point $(\lambda^*, 0, 0)$, where $\lambda^* := a(0)\,\lambda_1$ we will apply Theorem 4.1 of [3]. Furthermore, we prove results of non-existence and a priori bounds of coexistence states of (P) in cases where $b > 0$ and $b < 0$.

Joint work with Antonio Suárez (Universidad de Sevilla) and Cristian Morales-Rodrigo (Universidad de Sevilla).

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A priori estimates for solutions of $g-$Laplace type problems

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Abstract. In this work we study a priori bounds for weak solutions to elliptic problems with non-standard growth that involve the so-called $g-$Laplace operator. The $g-$Laplacian is a generalization of the $p-$Laplace operator that takes into account different behaviors than pure powers. The method to obtain this a priori estimates is the so called "blow-up" argument developed by Gidas and Spruck. Finally, we applied this a priori bounds to show some existence results for these problems.

Joint work with Julían Fernández Bonder (UBA) and Analía Silva (UNSL)

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On a subspace of the Beppo-Levi space $\mathcal{D}^{1,p}(\mathbb{R}^N_+)$ and applications

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Abstract. In this work we are concerned with embeddings results for a space of functions defined in the half-space. We obtain some Sobolev embeddings, Trudinger-Moser inequalities that can be used to show existence results for some classes of problems in the half-space.

Joint work with Everaldo Souto de Medeiros (UFPB) and João Marcos Bezerra do Ó (UFPB)

Nonautonomous Schrödinger equations with inhomogeneous nonlinearity

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Abstract. We will present the existence of positive ground or bound state solutions for a class of nonlinear Schrödinger equations:

$$-\Delta u + \lambda u = a(x)f(u), \qquad u \in H^1(\mathbb{R}^N),\ N \geq 3,$$

where the function a is a positive function, symmetric under some group action G. The inhomogeneous nonlinearity f, under very mild assumptions, is asymptotically linear or superlinear and subcritical at infinity, with $f(s)/s$, $s > 0$, not satisfying any monotonicity condition.

Joint work with Liliane de Almeida Maia (UnB) and Ricardo Ruviaro (UnB).

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Quasilinear elliptic problems with exponential growth via the Nehari manifold method

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Abstract. In this work we will be concerned with the problem

$$-\Delta u - \frac{1}{2}\Delta(a(x)u^2)u + V(x)u = f(u), \quad x \in \mathbb{R}^2,$$

where V is a potential continuous and $f : \mathbb{R} \to \mathbb{R}$ is a superlinear continuous function with exponential subcritical or exponential critical growth. We use as a main tool the Nehari manifold method in order to show existence of nonnegative solutions and existence of nodal solutions. Our results complement the classical results due to Jia-quan Liu, Ya-qi Wang and Zhi-Qiang Wang in [1].

Joint work with Giovany M. Figueiredo (UnB) and Ricardo Ruviaro (UnB).

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Existence and regularity of the solution of a semilinear elliptic equation problem with singular nonlinearity

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Abstract. In this work we will show the existence and regularity of the solution of problem (1), following the studies of Lucio Boccardo and Luigi Orsina [1],

$$\begin{cases} -\text{div}(M(x)\nabla u) = \frac{f(x)}{u} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \partial\Omega \end{cases} \tag{3}$$

in this case, $\Omega \subset \mathbb{R}^N$ is a bounded open, with $N \geq 2$, f is either a nonnegative function belonging to some Lebesgue space and M is a bounded elliptic matrix. To discuss the problem, we use the method of approximation and results such of Maximum Principle, Schauder's fixed point theorem and estimates produced in [2].

Joint work with Luis Henrique de Miranda (UnB).

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Trudinger-Moser and Adams type inequalities on weighted Sobolev spaces and applications

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Abstract. In this talk, we will explore sharp Sobolev embeddings and Adams-type inequalities on a class of Sobolev spaces with potential weights, without assuming boundary conditions. We will focus on the borderline Sobolev embedding into the exponential class, highlighting its sharp constant. By applying these concepts, we will demonstrate the existence of nontrivial solutions for elliptic equations with nonlinearities including both polynomial and exponential growths.

Additionally, we will investigate embeddings on a class of Sobolev spaces with potential weights on unbounded domains. Our findings will reveal embeddings into weighted Lebesgue spaces, specifically L^q_θ, using radial power weights. Based on these embeddings, we will investigate the existence and non-existence of maximizers for Trudinger-Moser type inequalities. Furthermore, we will present a technique to enhance the maximal integrability by selectively removing necessary terms from the exponential series while maintaining the continuity of the embedding..

Joint work with João Marcos Bezerra do Ó (UFPB) and Guozhen Lu (University of Connecticut)

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Existence and limit behavior of least energy solutions to constrained Schrödinger-Bopp-Podolsky systems in $\mathbb{R}^3$

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Abstract. Consider the Schrödinger–Bopp–Podolsky system in $\mathbb{R}^3$ under an L^2-norm constraint,

$$\begin{cases} -\Delta u + \omega u + \phi u = u|u|^{p-2}; \\ -\Delta \phi + a^2\Delta^2\phi = 4\pi u^2; \\ \|u\|_{L^2} = \rho, \end{cases} \tag{SBP$_{a,\rho}$}$$

where $a, \rho > 0$ are fixed and our unknowns are $\omega \in \mathbb{R}$ and $u, \phi\colon \mathbb{R}^3 \to \mathbb{R}$. As for the existence of solutions, we prove that if $2 < p < 3$ (resp., $3 < p < 10/3$) and $\rho > 0$ is sufficiently small (resp., sufficiently large), then (SBP$_{a,\rho}$) admits a least energy solution. These results are followed by a proof that, up to translation, least energy solutions are radially symmetric when $2 < p < 14/5$ and $\rho > 0$ is sufficiently small. Under the same hypotheses, we show that as $a \to 0$, least energy solutions converge up to subsequence to a least energy solution of the constrained Schrödinger–Poisson–Slater system,

$$\begin{cases} -\Delta u + \omega u + \phi u = u|u|^{p-2}; \\ -\Delta \phi = 4\pi u^2; \\ \|u\|_{L^2} = \rho. \end{cases} \tag{SPS$_{\rho}$}$$

Joint work with Gaetano Siciliano (USP).

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Regularizing Effect for a Class of Maxwell - Schrödinger Systems

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Abstract. In this paper we prove the existence and regularity of weak solutions for the following system

$$\begin{cases} -\text{div}(M(x)\nabla u) + h(x,u,v) = f \ \text{ in } \ \Omega \\ -\text{div}(M(x)\nabla v) = g(x,u,v) \ \text{ in } \ \Omega \\ \quad u = v = 0 \ \text{ on } \ \partial\Omega, \end{cases}$$

where Ω is an open bounded subset of $\mathbb{R}^N$, for $N > 2$, $f \in L^m(\Omega)$, with $m > 1$ and h, g are two Carathéodory functions. We prove that under appropriate conditions on h and g there exist solutions which escape the predicted regularity by classical Stampacchia's theory, the so-called regularizing effect.

Joint work with Luís Henrique de Miranda (UnB).

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On the Liouville Property for degenerate Fully Nonlinear Equations

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Abstract. In this paper, we obtain Liouville-type results to fully nonlinear second-order degenerate equations. As a consequence, using degree theory, we obtain existence results for superlinear Dirichlet problems

Joint work with Disson S. dos Prazeres (UFS) and Jose Anderson V. Cardoso (UFS).

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Regularity In Diffusion Models With Gradient Activation

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Abstract. We establish sharp regularity estimates for solutions of highly de- generate fully nonlinear elliptic equations. These are free boundary models in which a nonlinear diffusion process drives the system only in the region where the gradient surpasses a given threshold. Our main result concerns the existence of a universal modulus of continuity for Du, up to the free boundary. We provide some applications of the methods with other treads of research.

Joint work with D. Araújo (UFPB) and E. Teixeira (UCF)

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Logistic equation with Robin type boundary conditions and undefined coefficients

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Abstract. In this work, we follow the paper of Umezu [1] and we study existence and uniqueness of solutions for the following stationary logistic equation

$$\begin{cases} -\Delta u = \lambda(g(x) - cu)u & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \eta} = \lambda h(x)u & \text{on } \partial\Omega, \end{cases} \tag{4}$$

where Ω is a bounded domain of $\mathbb{R}^N$, $N \geq 2$, with smoth boundary $\partial\Omega$, λ is a real parameter, $g \in C^\theta(\overline{\Omega})$ e $h \in C^{1+\theta}(\partial\Omega)$, $0 < \theta < 1$, both functions can change sign, c is a non-negative constant and $\eta = \eta(x)$ denotes the unit exterior normal at $x \in \partial\Omega$. We emphasize that the equation (4) arising from a population dynamics model and, therefore, we are interested in searching positive solutions for $\lambda 0$. Using tools from variational method (Lagrange Multipliers and Minimization), we study the eigenvalue problem, that is, the problem (4) with $c = 0$. Then, applying the sub and supersolution methods we establish the existence, non-existence and uniqueness of the positive solutions for (4) with $c > 0$. Finally, we prove a priori bounds and study the asymptotic behavior of the solutions with respect to λ.

Joint work with Willian Cintra da Silva (UnB).

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