Abstracts

Detecting hypercentrality from small subgroups

[21] (2009)

Abstract

We show that the nilpotency of the metacyclic subgroups of a hypercyclic group is already sufficient to conclude the hypercentrality of the whole group. We also construct a center-free metabelian group, all of whose polycyclic subgroups are abelian.

The quasi-superfluous elements of a group

[20] (2009)

Abstract

An element x of a group G is *quasi-superfluous* in G if $\langle x \rangle$ permutes with every maximal subgroup of G. The set $\mathbf{Qs}(G)$ of all quasi-superfluous elements of G (a set which contains the Frattini subgroup $\Phi(G)$) is known to be a nilpotent characteristic *subgroup* in every finite group. In the article, some sufficient conditions for the subgroup property of $\mathbf{Qs}(G)$ are established.

Properties of soluble groups detectable at their metabelian subgroups

[19] (2009)

Abstract

If in a soluble group G, the metabelian subgroups have finite (or polycyclic, or Chernikov) quotients by their centers, then so does G.

On the local hypercenter of a group

[18] (2008)

Abstract

We introduce a *local hypercenter* of an arbitrary group and study its basic properties. With this concept, it turns out that classical theorems of Baer, Mal'cev and McLain on locally nilpotent groups can be obtained as special cases of statements which are valid in any group. Furthermore, we investigate the connection between the local hypercenter of a group and the intersection of its maximal locally nilpotent subgroups.

Property preserving subgroups of a group

[17] (2006)

Abstract

Motivated by some characterizations of the hypercenter of a finite group, we investigate in every group G, with respect to any subgroup closed group class $\mathfrak{X}$, its $\mathfrak{X}$-preserver $\Pi_{\mathfrak{X}}(G)$ - a canonical characteristic subgroup, which coincides with the hypercenter of G when $\mathfrak{X}=\mathfrak{N}$ is the class of nilpotent groups and G is finite. We study general properties of $\Pi_{\mathfrak{X}}(G)$, discuss special properties and show the utility of the introduced concepts in the case of concrete classes.

The Dietzmann property of some classes of groups with locally finite conjugacy classes

[16] (2004)

Abstract

A subgroup- and quotientgroup closed class $\mathfrak{D}$ of groups is a *Dietzmann class* if the normal closure $\langle x^G \rangle$ of an element x of an arbitrary group G is a $\mathfrak{D}$-group, provided that $\langle x \rangle \in \mathfrak{D}$ and G induces on $\langle x^G \rangle$ a $\mathfrak{D}$-group of automorphisms. For a set π of prime numbers, let $\mathfrak{F}_\pi$ denote the class of finite, $L\mathfrak{F}_\pi$ that of locally finite π-groups. For any subgroup- and quotientgroup closed class $\mathfrak{X}$ with $\mathfrak{F}_\pi \subseteq \mathfrak{X} \subseteq L\mathfrak{F}_\pi$, let $H\mathfrak{X}$ denote the class of hyper-$\mathfrak{X}$-groups, $(H\mathfrak{X})C$ that of groups with $H\mathfrak{X}$-conjugacy classes. We show that $H\mathfrak{X}$ and $(H\mathfrak{X})C$ - in particular $H\mathfrak{F}_\pi$, $(H\mathfrak{F}_\pi)C$ and $(L\mathfrak{F}_\pi)C$ - are Dietzmann classes.

When is a polycyclic-by-finite group hypercentral-by-finite?

[15] (1999)

Abstract

To conclude that a polycyclic-by-finite group is hypercentral-by-finite, it is sufficient to test the same property at its nilpotent-by-cyclic subgroups only.

Polycyclic-by-finite groups and strong Carter subgroups

[14] (1999)

Abstract

Based on the theory of Carter subgroups of finite solvable groups, a series of characterizations of the finite-by-nilpotent among the polycyclic-by-finite groups is established.

$\mathfrak{X}$C-elements in groups and Dietzmann classes

[13] (1999)

Abstract

Group classes $\mathfrak{D}$ are investigated for which in every group G the following implication is generally true :

$$G/\mathbf{C}_G(x^G) \in \mathfrak{D} \quad \text{and} \quad \langle x \rangle \in \mathfrak{D} \quad \Longrightarrow \quad \langle x^G \rangle \in \mathfrak{D}.$$

Here $G/\mathbf{C}_G(x^G)$ is the automorphism group induced by G on the normal subgroup $\langle x^G \rangle$ of G generated by the element $x \in G$. Specialized for the class $\mathfrak{D} = \mathfrak{F}$ of all finite groups, this implication is known as *Dietzmann's Lemma.*

Hypercentral embedding and pronormality

[12] (1998)

Abstract

Hypercentral embedding of a subgroup H of a finite group G is equivalent with the permutability of H with all pronormal subgroups of G.

(H is *hypercentrally embedded* in G if H/H_G is contained in the hypercenter - the last term of the upper central series - of G/H_G, where H_G the intersection of all conjugates of H in G. H is *pronormal* in G, if for every $g \in G$, H is conjugate to H^g under $\langle H, H^g \rangle$.)

The p-Huppert-subgroup and the set of p-quasi-superfluous elements in a finite group

[11] (1993)

Abstract

A theorem of B. Huppert characterizes the p-supersolvable among the p-solvable groups by certain index properties of their maximal subgroups. Based on this theorem, a prime-number-parametrized family of canonical characteristic subgroups $\mathbf{\Gamma}_p(G)$ - the p-Huppert subgroups of G - and their intersection $\mathbf{\Gamma}(G)$ is introduced in every finite group G, in such a way that $\mathbf{\Gamma}_p(G)$ (resp. $\mathbf{\Gamma}(G)$) coincides with G when G is p-supersolvable (resp. supersolvable). With these concepts, Huppert's characterizations of supersolvable and p-supersolvable groups become theorems which are valid in any finite group G. Special interest is dedicated to a description of $\mathbf{F}_p(\mathbf{\Gamma}_p(G))$, the largest p-nilpotent normal subgroup of $\mathbf{\Gamma}_p(G)$ and of $\mathbf{F}(\mathbf{\Gamma}(G))$, the Fitting subgroup of $\mathbf{\Gamma}(G)$ by means of permutability properties of their cyclic subgroups. For example: $\mathbf{F}(\mathbf{\Gamma}(G))$ coincides with the set of all elements x in G such that $\langle x \rangle$ is permutable with all maximal subgroups of G. Since the elements of the Frattini subgroup $\Phi(G)$ (= the set of the superfluous elements) have trivially this property, $\mathbf{F}(\mathbf{\Gamma}(G))$ can be seen as the set of the "quasi-superfluous" elements of G.

A completeness property of certain formations

[10] (1992)

Abstract

Saturated formations which contain the class of all supersolvable groups are closed under subgroup products $G = AB$, if every subgroup of A is permutable with every subgroup of B. Subgroups with this property are called *totally permutable.*

Zur Vertauschbarkeit und Subnormalität von Untergruppen

[9] (1989)

Abstract

Properties of M-embedded subgroups X of a finite group G are investigated, where M-embedding means that X is permutable with all *maximal* subgroups of G. Among several other results (e.g. supersolvability of a solvable group is equivalent with the M-embedding of all its subnormal subgroups) it is shown that *Nilpotency of a finite group X can be characterized by the property that nilpotent M-embedded subgroups X are always subnormal in G, whereas every non-nilpotent group X can be non-subnormally M-embedded into a bigger group G.*

A note on subnormality in factorizable finite groups

[8] (1984)

Abstract

It is shown that, if a subgroup X of a solvable factorized finite group $G = AB$ is such that X is subnormal in $\langle X, g\rangle$ for all $g \in A \cup B$, then X is subnormal in G.

Um problema da teoria dos subgrupos subnormais

[7] (1977)

Abstract

If a solvable subgroup X is contained as a subnormal subgroup in two subgroups A and B of a finite group G which permute, then X is also subnormal in $AB = BA$. It is conjectured that the solvability of the subgroup X is unnecessary for this conclusion. This conjecture is completely solved for finite groups in

WIELANDT, H. Subnormalität in faktorisierten endlichen Gruppen. Journal of Algebra **69**, 305-311, 1981.

Faktorisierte p-auflösbare Gruppen

[6] (1976)

Abstract

A p-solvable finite group ($p > 2$) written as a product of pairwise permutable subgroups has p-length at most 1, if it is possible to find in the Sylow p-subgroups of each of the factors a subnormal series of length $\leq \frac{p-1}{2}$ with cyclic quotients.

Über die 2-Nilpotenz faktorisierbarer endlicher Gruppen

[5] (1976)

Abstract

A finite group is 2-nilpotent, whenever it can be written as the product of pairwise permutable subgroups, each of which is a direct product of a cyclic 2-group by a group of odd order.

This theorem is an extension of (and uses) the classical Burnside result which guarantees the 2-nilpotency of a finite group if its Sylow 2-subgroups are cyclic.

Bemerkung zu einem Satz von Huppert

[4] (1975)

Abstract

A theorem of B. Huppert states that a finite group which is the product of pairwise permutable cyclic subgroups, is supersolvable. In this article it is shown that, if G is the product of pairwise permutable subgroups, the Sylow subgroups of which are cyclic and which all have a normal Sylow 2-subgroup, then G has a Sylow tower - with the prime numbers ordered opposite to the natural order - and G is p-supersolvable for all Fermat primes. If G has odd order, then G is supersolvable.

The embedding of quasinormal subgroups in finite groups

[3] (1973)

Abstract

If Q is a quasinormal subgroup of a finite group G (= a subgroup permutable with every subgroup of G), then Q is hypercentrally embedded in G.

(The necessary and sufficient permutability property for the hypercentral embedding of a subgroup H of G is given in the article

[12] **Hypercentral embedding and pronormality** (1998)

Endliche metanilpotente Gruppen

[2] (1972)

Abstract

Some characterizations of finite solvable groups of Fitting length ≤ 2 are established. Equivalent to solvability of Fitting length ≤ 2 of a group G are:

a) G is a product of two nilpotent subgroups and the p-length of G is ≤ 1 for all primes p.

b) The nilpotent residuals of all maximal subgroups of G permute with all maximal subgroups of G.
c) The nilpotent residuals of all maximal subgroups of G are subnormal in G.

Also: Sufficient for metanilpotency of a product of two nilpotent subgroups is the *modularity* of the factors.

Normality conditions for quasinormal subgroups in finite groups

[1] (1971)

Abstract

A complete classification of the finite groups G with the following property is given: There exists a quasinormal non-normal subgroup in G, but in all proper subgroups and in all proper quotient groups of G, quasinormality coincides with normality.

(this article is an extract from the author's doctoral thesis)